MOLPP FRAME WORK FOR A MANUFACTURING INDUSTY: A MATRIX BASED OPTIMIZATION APPROACH

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ABSTRACT

This study discusses on solving a multi-objective linear programming (MOLP)problem for a PVC door manufacturing company**.** The company has set three objectives namely Maximization of profits, Minimizing of production costs, Efficient use of available resources such as labour, materials, and time. To achieve these objectives, the problem is formulated using a system of linear equations, and a matrix inversion method is applied to find the optimal levels for different types of PVC doors. A numerical example is provided to demonstrate and offer a balanced production plan that meets all three objectives effectively, showing that this approach is useful for decision-making in the PVC door manufacturing industry.The results are compared with the existing methods for a better approach.

KEYWORDS

LPP, MOLPP, Echelon Method,Frame work for a manufacturing industry, Decision Making.

1 INTRODUCTION

In today's competitive industrial environment,manufacturing industries are expected to meet growing demands while maintaining efficiency, profitability and sustainability. Decision-making in such industries is complex because managers rarely deal with a single objective. Instead,they must achieve several goals simultaneously-such as minimizing production costs, maximizing profit,reducing resources wastage, maintaining product quality,meeting delivery deadlines,and ensuring customer satisfaction.These multiple requirements often conflict with one another, making the problem more challenging.Traditionally, Linear programming problem (LPP) has been used to optimize a single objective function, such as cost minimization or profit maximization, subject to a set of constraints. While effective in simple cases,this approach is inadequate in practical industrial environments where trade-offs between different objectives are avoidable.For example, minimizing cost may negatively affect product quality,or maximizing production may lead to increased use of resources and higher operational costs.

To overcome this limitation, Multi-objective linear programming problem (MOLPP) has emerged as an advanced extension of linear programming. MOLPP allows decision makers to optimize more than one objective function simultaneously under a common set of constraints.These solutions represent situations where improvement in one objective cannot be achieved without compromising another. This gives managers a clear understanding of the trade-offs andenables them to select the most appropriate solution according to their priorities and preferences.

In the framework of manufacturing industries, MOLPP plays a vital role in addressing diverse challenges. It can be applied to production planning, where industries must determine the optimal product mix to balance cost,profit,and labour utilization. It is also useful in scheduling problems,where objectives such as minimizing machine idle time, meeting deadline, and reducing overtime must be considered together. In supply chain management, MOLPP helps balance transportation costs, delivery times, and service quality. In addition, it can support sustainable manufacturing by integrating environmental objectives such as reducing emissions and energy consumption alongside economic goals.

Another important feature of MOLPP in manufacturing is its ability to incorporate managerial preferences. Since decision-makers may value some objectives more than others, MOLPP models can be designed goal programming, or other approaches to reflect organizational priorities.This flexibility makes MOLPP a powerful tool for real world application.

Moreover, manufacturing industries operates in environments characterized by uncertainty and competition. MOLPP provides a structured mathematical approach to handle these complexities. By offering a range of trade-off solutions, it not only improves operational efficiency but also supports long-term strategic decision making. Companies can evaluate and select solutions that ensure competitiveness, customer satisfaction, and sustainable growth.

In summary, Multi-Objective Linear Programming provides a Comprehensive optimization framework for manufacturing industries. It extends beyond the limitations of single objective models and enables industries to consider cost, quality, time, and resource utilization together. Thus, MOLPP serves as a crucial decision making tool in modern manufacturing systems, fostering productivity, efficiency, and competitiveness in an increasingly complex and dynamic market environment.

2 Review of Literature

Gaurav Sharma [8] in 2015 addressed the limitations of traditional transportation problem by considering both transportation cost and transportation time as simultaneous objectives. The study formulated a multi-objective transportation problem(MOTP) and proposed a method to convert the two conflicting objectives into a single composite function using weighting or goal programming techniques. The paper contribution lies in demonstrating a practical and effective way to optimize transportation system where cost-time trade-offs are critical in decision making.

Belenson&Kapur[3] in 1973 introduced one of the earliest algorithms for multi-objective linear programming, focusing on generating efficient solutions through systematic LP-based procedures. Annamadas& Rao [1] in 2009 advanced the field by combining game theory with particle swarm optimization, providing an effective metaheuristic approach for complex engineering optimization problems. Batamizet al.[2] in the year 2020 extended MOLP to interval environment, proposing methods to obtain efficient solutions when data are uncertain or imprecise. Biswal & Acharya (2013) contributed by formulation multi-choice multi-objective linear programming, addressing problems where decision variables have discrete choice levels [1-4].

Chandra & Mehta [5] in 2018 illustrated the application of multi-objective linear programming (MOLP) in production planning, optimizing conflicting goals such as cost, time, and resources utilization to support practical decision making. Charkhgard, Savelsbergh, and Talebian[6]in 2018 developed a linear-programming based algorithm to solve optimization problems with multi-linear objective functions and affine constraints, providing an efficient method for handling complex multi-objective formulations. Cohon[7] in 1978 laid the foundational theory of multi-objective programming and planning, introducing systematic procedures and goal- setting approaches for decision making and resources allocation. Collectively, these works reflect the progression of MOLP from theoretical foundations to advanced algorithms and real-worldapplications. [5-7].

Gupta & Sharma [9] in 2020 highlighted the importance of linear programming in manufacturing industries, showing how it improves production efficiency,resources allocation, and cost minimization. Gupta & Dutta [10] in 2014 provided an overview of MOLP and its wide industrial applications, explaining how MOLP helps handle conflicting objectives in areas such as production planning, Scheduling, and decision making [9-10].

Hanowski, Perez and Dingus [11] in 2005 examined driver distraction among long-haul truck drivers and highlighted how human factor and behavioural issues impact safety and operational performance in transportation system. Hillier and Lieberman [12] in 2015 in their widely used operations research textbook, provided comprehensive methods and analytical tools for optimization, decision making, and system analysis, forming a strong theoretical base for solving real-world operational problems. Hwang and Masud [13] in 1979presented foundational concepts in multiple objective decision making, offering systematic techniques for handling conflicting criteria in complex decision environments.[11-13].

Jain and Singh[14] in 2017 discussed various MOLP techniques used in production optimization and showed how multi-objective models help improve efficiency, resource use, and decision making in manufacturing. Miettinen [15] in 1999 provided a comprehensive foundation for non-linear decision making, explaining key concepts, solution methods, and theoretical principles for handling complex, conflicting objectives. Mohamed and Hasen[16] in 2013 reviewed different matrix based methods in optimization, highlighting their usefulness in simplifying computations, improving solution accuracy, and supporting analytical problem solving.[14-16].Ponnamabalamet al.[17] in 2000 proposed a matrix based method to solve multi objective problems in flexible manufacturing system, helping to manage multiple goals more efficiently. Rehman and Ahmed [18] in 2021 applied multi-objective linear programming to improve resources allocation in industries by balancing cost, time and resources use. Rivaz and Yaghoobi [19] in 2013 addressed uncertainty in multi-objective problem by using a minimax regret approach, which provides reliable solutions when objective values are not exact.[17-19].Sinha and Mukhopadhyay[20] in 2014 explored the use of scalarization methods to solve multi-objective linear programming problems by converting several objectives into a single optimized form. Their work showed that scalarization is an effective way to handle conflicting goals and generate a set of efficient solutions. Sundaresan and Ananthanarayanan[21] in 2016 introduced a matrix-based techniques for solving multi-objective linear programming problems in manufacturing, offering a systematic and organized way to analyze multiple criteria at once.Their approach helped improves solution accuracy and made complex manufacturing decisions easier to manage.[20,21]Zeleny[23] in 1982 provided a strong theoretical foundation for multiple criteria decision making by explaining why real-life decisions often involves several conflicting objectives and how mathematical models can help compare alternatives. His work clarified key ideas such as trade-offs, efficiency and optimal choice in multi-criteria situations.Later,Tabucanon [22] in 1988 built on this theory by focusing more on practical industrial applications, showing how multiple criteria decision-making methods can be used to improves decisions in areas like production planning, scheduling and resource management. [22,23]. Applications of MOLP in manufacturing show its effectiveness in product mix planning, resources allocation, and scheduling. Research demonstrates that MOLP not only improves profitability but also ensures efficient use of materials, labour, and machine time. Moreover,literature indicates that using MOLP framework helps industries adapt to competitive markets, reduce waste, and achieve sustainable operations.

Thus, the review of earlier studies confirms that MOLP, especially when combined with matrix-based optimization techniques, offers a reliable and flexible tool for modern manufacturing industries to achieve multiple goals in a balanced manner.

3 PRELIMINARIES

3.1 PROFIT AND LOSS

In Linear Programming Problems (LPP), profit is represented by the objective function. This function is a linear expression that aims to be maximized (or minimized) subject to certain constraints. It reflects the quantity we want to optimize, like maximizing profit or minimizing cost, and is directly influenced by the decision variable.

3.2 MAXIMUM AND MINIMUM COST

Linear Programming states that the maximum (or minimum) value of the objective function always takes place at the vertices of the solution.

3.3 SET UP COST

This is the cost associated with the setting up of machinery before starting production. Set-up cost is generally assumed to be independent of the quantity ordered for or produced

3.4 ORDERING COST

This is a cost associated with ordering of raw materials for production purposes.

3.5 PURCHASE COST

The cost of purchasing (or producing) a unit of an item is known as purchase cost .

3.6 CARRYING(OR HOLDING) COST

This cost generally includes the costs such as rent for space used for storage etc.

4 MATRIX FORM OF MULTI OBJECTIVE LINEAR PROGRAMMING PROBLEM

A Mathematical model of MOLPP with objective can be written as:

Minimize or Maximize $Z_r = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{r}.x_{ij}$ r = 1,2,3,……….k

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i , \quad i = 1,2, \dots \dots , m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2, \dots \dots . , n$$

$$x_{ij} \geq 0 \ \forall \ i, j$$

The linear programming problem can be expressed in the matrix form as follows

Maximize(or minimize)$Z = CX$

Subject to,

AX ($\leq = \geq$ b)(constraints)

$X \geq 0$ (Non-negativityconstraint)

Where, $X = (x_1, x_2, x_3, \dots\dots x_n), C = (c_1, c_2, c_3, \dots\dots c_n)$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}, \qquad A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \cdot & \cdot & a_{mn} \end{bmatrix}$$

5 Algorithm for the Matrix Formulation of linear programming problem

Step 1: Compute the row reduced echelon form of augmented matrices (a_1, b_1), (a_2, b_2), (a_3, b_3) by applying suitable row operation, they may be following three case:

Case1: If rank $(a_1) \neq$ rank (a_1, b_1) or rank $(a_2) \neq$ rank (a_2, b_2) or rank $(a_3) \neq$ rank (a_3, b_3) then LPP is inconsistent and no non- negative solution exists.

Case2: If rank $(a_1) =$ rank (a_1, b_1) and rank $(a_2) =$ rank (a_2, b_2) and rank $(a_3) =$ rank (a_3, b_3) but there exist at least one negative element in the $(n+1)^{th}$ column of the row reduced echelon form of augmented matrices (a_1, b_1) or (a_2, b_2) or (a_3, b_3) then the system of the equation are inconsistent and no non-negative solution exists.

Case3: Ifrank $(a_1) =$ rank (a_1, b_1) and rank $(a_2) =$ rank (a_2, b_2) and rank $(a_3) =$ rank (a_3, b_3) and all the element of the $(n+1)^{th}$ column of the row reduced echelon form of augmented matrices (a_1, b_1), (a_2, b_2), (a_3, b_3) are nonnegative then the system of equation are consistent (i.e.) non-negative solution exists.

Step 2: Compute the value of x_i, y_i, z_i using the row reduced echelon form of augmented matrices (a_1, b_1), (a_2, b_2) and (a_3, b_3) respectively.

6 NUMERICAL EXAMPLE

A manufacturing company produces PVC doors and the production process involves three sequential stages—**Fabrication, Door Design, and Packing**. Each stage contributes to both the profit and cost of production and consumption of resources such as labor, materials, and time. The goal is to optimize the production process by maximizing total profit and minimizing the associated production costs within the available resource limits.Theproblem is formulated as a Linear Programming Problem (LPP) using the given profit, cost and resource consumption data for each stage. This includes defining an objective function and setting appropriate constraints based on labor, material, and time availability, along with ensuring non-negativity conditions for the decision variables.A company manufacturing PVC doors undergoes different process to obtain the final product. The production includes three Continuous stages areFabrication, door design, and Packing. Each stage gives a profit per unit of Fabrication is a ₹3, Door design ₹5, Packing is ₹4 and also the product cost in Fabrication is ₹2, Doon design is ₹3 and packing is ₹1.Tomanufacture the doors, the following resources are consumed. Formulatea LPP to maximize profit and minimize production cost.

Solution:

Let x_1: denotes the number of units processed through Fabrication

- x_2:denotesthe number of units processed through Door Design
- x_3: denotes the number of units processed through Packing

Resource Availability

- Labours: 100 nos
- Material: 120 kg
- Time: 90 hours

Labour Constraint

The total labours required for all stages of production should not exceed 100.

- Each unit of Fabrication requires 2 labours.
- Each unit of Door Design requires 3 labours.
- Each unit of Packing requires 2 labours.

Material Constraint

The total material used in all stages must not exceed 120 kilograms.

- Each unit of Fabrication uses 4 kg of material.
- Each unit of Door Design uses 2 kg of material.
- Each unit of Packing uses 3 kg of material.

Time Constraint

The total time required for all operations should not exceed 90 hours.

- Each unit of Fabrication takes 3 hours of processing time.
- Each unit of Door Design takes 2 hours.

- Each unit of Packing takes 1 hour.

Process Continuity Constraint

- Since the production process is continuous (a unit must pass through all three stages),
- The number of units processed in Fabrication, Door Design, and Packing must be the same:
- Fabrication units = Door Design units = Packing units

Non-Negativity Constraint

The number of units produced in each stage must be greater than or equal to zero:

Fabrication ≥ 0, Door Design ≥ 0, Packing ≥ 0

Solution by matrix inversion method

Step 1: The multi objective linearprogramming problem with two objective function is given by

Max Z = $3x_1 + 5x_2 + 4x_3$

Min Z = $2x_1 + 3x_2 + x_3$

Subject to constraints ,

$$2x_1 + 3x_2 + 2x_3 \leq 100$$

$$4x_1 + 2x_2 + 3x_3 \leq 120$$

$$3x_1 + 2x_2 + x_3 \leq 90$$

$$x_1\, x_2 x_3 \geq 0$$

Step2:Considered the system of linear equationas

$$2x_1 + 3x_2 + 2x_3 = 100$$

$$4x_1 + 2x_2 + 3x_3 = 120$$

$$3x_1 + 2x_2 + x_3 = 90$$

$$x_1, x_2, x_3 \geq 0,$$

Step3:The system of linear equationcan be written into the matrix form as

$$\text{Coefficient Matrix A} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\text{Constant Matrix B} = \begin{bmatrix} 100 \\ 120 \\ 90 \end{bmatrix}$$

$$\text{Variable Matrix X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Step:4 The matrix $[A, \quad I] = \left[\begin{array}{ccc|ccc} 2 & 3 & 2 & 1 & 0 & 0 \\ 4 & 2 & 3 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$

$$= \left[\begin{array}{ccc|ccc} 2 & 3 & 2 & 1 & 0 & 0 \\ 4 & 2 & 3 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] R_1 \rightarrow \frac{R_1}{2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 1 & \frac{1}{2} & 0 & 0 \\ 4 & 2 & 3 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] R_2 \rightarrow R_2 - 4R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 1 & \frac{1}{2} & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] R_3 \rightarrow R_3 - 3R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 1 & \frac{1}{2} & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & \frac{-5}{2} & -2 & \frac{-3}{2} & 0 & 1 \end{array}\right] R_3 \rightarrow 2R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 1 & \frac{1}{2} & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -5 & -4 & -3 & 0 & 2 \end{array}\right] R_3 \rightarrow 4R_3 - 5R_2$$

$$= \left[\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 1 & \frac{1}{2} & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & -11 & -2 & -5 & 8 \end{array}\right]$$

Step: 5 By solving the matrix $\begin{bmatrix} 1 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & -4 & -1 & -2 \\ 0 & 0 & -11 & -2 \end{bmatrix}$ We get, $\begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \\ \frac{11}{2} \\ \frac{}{11} \end{bmatrix}$

By solving the matrix $\begin{bmatrix} 1 & \frac{3}{2} & 1 & 0 \\ 0 & -4 & -1 & 1 \\ 0 & 0 & -11 & -5 \end{bmatrix}$ We get, $\begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} \frac{1}{11} \\ \frac{-4}{11} \\ \frac{5}{11} \end{bmatrix}$

By solving the matrix $\begin{bmatrix} 1 & \frac{3}{2} & 1 & 0 \\ 0 & -4 & -1 & 0 \\ 0 & 0 & -11 & 8 \end{bmatrix}$ We get, $\begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} \frac{5}{11} \\ \frac{2}{11} \\ \frac{-8}{11} \end{bmatrix}$

- ❖ If the Coefficient Matrix A is non-singular matrix , we proceed to solution.
- ❖ Determinant of coefficient matrix A is 11 i.e. A is non-singular matrix then we proceed to further solution.
- ❖ Inverse matrix of coefficient matrix A is

$$A^{-1} = \begin{bmatrix} 1 & \frac{1}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{-4}{11} & \frac{2}{11} \\ \frac{2}{11} & \frac{5}{11} & \frac{-8}{11} \end{bmatrix}$$

After applying the matrix inversion method we get the value i.e.$x_1 = 151.81$; $x_2 = 18.18$; $x_3 =$ -25.45 with this value we get the optimum solution of objectives functions are

$Z_1 = 444.53$ and $Z_2 = 332.71$

7 CONCLUSION

The matrix inversion method is very simple from the computational point of view and also easy to understand and apply. In this method, we obtain a sequence of optimal solution to a MOLPP for a sequence of various time intervals. This method provides a set of optimum solution to MOLPP which help the decision makers to take an appropriate decision, depending on his financial position and the decision maker to evaluate the economical activities and make the correct managerial decision.

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