Formulation of dio-quadruple with property $D(k^2+1)$

S. Vidhyalakshmi[1], S. Aarthy Thangam[2*], M.A. Gopalan[3]

[1,3]*Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.*

[2*]*Research Scholar, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.*

[1]vidhyasigc@gmail.com

[2*]aarthythangam@gmail.com

[3]mayilgopalan@gmail.com

***Abstract*— This paper concerns with the problem of constructing dio-quadruple $(1, n, c_s, c_{s+1})$ such that the product of any two members of the set subtracted by their sum and added with k^2+1 is a perfect square.**

***Keywords*— Dio-Quadruples, Pell equation, Integer solutions.**

I. INTRODUCTION

A set of m distinct positive integers $\{a_1, a_2, a_3, ...a_m\}$ with $a_i a_j \pm (a_i + a_j) + n$ as a perfect square for all $1 \le i < j \le m$ is called a Special Dio m-tuple with property $D(n)$. In [1-7], problems on special dio-quadruples with suitable properties are analysed. This motivated us to construct sequences of special dio-quadruples with property $D(k^2+1)$.

II. METHOD OF ANALYSIS

Let $a = 1$ and $b = n$ be two integers such that $ab - (a+b) + k^2 + 1$ is a perfect square. Therefore (a, b) is the special dio-2-tuple with property $D(k^2+1)$.

Let c be any non-zero integer such that

$$ac - (a+c) + k^2 + 1 = p^2 \tag{1}$$

$$bc - (b+c) + k^2 + 1 = q^2 \tag{2}$$

Note that (1) is satisfied automatically.

$$(2) \Rightarrow \quad (n-1)c + k^2 - n + 1 = q^2 \tag{3}$$

which is satisfied by $c_0 = 1$, $q_0 = k$

Let (c_1, q_1) be the second solution of (3), where

$$c_1 = c_0 + h_0 \ , \ q_1 = h_0 - q_0 \tag{4}$$

in which h_0 is an unknown to be determined.

Substitution of (4) in (3) gives

$$h_0 = n - 1 + 2q_0 \tag{5}$$

In view of (4), we have

$$c_1 = n + 2q_0 \ , \ q_1 = n - 1 + q_0$$

The repetition of the above process leads to the general values of c and q satisfying (3) as

$$c_s = s^2 n - \left(s^2 - 1\right) + 2sk \;,\; q_s = s\left(n-1\right) + k \quad,\; s = 0,1,2,....$$

Thus $(1, n, c_s)$ is the special dio-3-tuple with property $D\left(k^2+1\right)$

Let c_{s+1} be any non-zero integer such that

$$n c_{s+1} - \left(n + c_{s+1}\right) + k^2 + 1 = \beta^2 \tag{6}$$

$$c_s\, c_{s+1} - \left(c_s + c_{s+1}\right) + k^2 + 1 = \gamma^2 \tag{7}$$

Eliminating c_{s+1} from (6) and (7), we obtain

$$\left(\left(n-1\right)s^2 + 2sk\right)\beta^2 - \left(n-1\right)\gamma^2 = k^2\left(\left(s^2-1\right)\left(n-1\right) + 2sk\right) \tag{8}$$

Introduction of the linear transformations

$$\beta = X + \left(n-1\right)T \;,\; \gamma = X + \left(\left(n-1\right)s^2 + 2sk\right)T \tag{9}$$

in (8) leads to

$$X^2 = \left(n-1\right)\left(\left(n-1\right)s^2 + 2sk\right)T^2 + k^2 \tag{10}$$

which is the pellian equation with the initial solutions $X_0 = \left(n-1\right)s + k \;,\; T_0 = 1$

In view of (9) and (6), we have

$$\beta_0 = \left(n-1\right)\left(\,s+1\right) + k$$

$$c_{s+1} = \left(n-1\right)\left(s+1\right)^2 + 2k\left(s+1\right) + 1 \;,\; s = 0,1,2,....$$

It is observed that the quadruple $(1, n, c_s, c_{s+1})$, $s = 0,1,2,3,....$ represents special dio-quadruple with property $D\left(k^2+1\right)$. A few numerical examples are presented in the Table: 1 below:

Table: 1 Numerical Examples

(n,k)	(a,b,c_0,c_1)	(a,b,c_1,c_2)	(a,b,c_2,c_3)	(a,b,c_3,c_4)	(a,b,c_4,c_5)
(1 , 1)	(1 , 1 , 1, 3)	(1 , 1 , 3 , 5)	(1 , 1 , 5 , 7)	(1 , 1 , 7 , 9)	(1 , 1 , 9 , 11)
(2 , 3)	(1 , 2 , 1 , 8)	(1 , 2 , 8 , 17)	(1 , 2 , 17 , 28)	(1 , 2 , 28 , 41)	(1 , 2 , 41 , 56)
(3 , 5)	(1 , 3 , 1 , 13)	(1 , 3 , 13 , 29)	(1 , 3 , 29 , 49)	(1 , 3 , 49 , 73)	(1 , 3 , 73 , 101)
(6 , 4)	(1 , 6 , 1 , 14)	(1 , 6 , 14 , 37)	(1 , 6 , 37 , 70)	(1 , 6 , 70 , 113)	(1 , 6 , 113 , 166)
(5 , 2)	(1 , 5 , 1 , 9)	(1 , 5 , 9 , 25)	(1 , 5 , 25 , 49)	(1 , 5 , 49 , 81)	(1 , 5 , 81 , 121)
(3 , 3)	(1 , 3 , 1 , 9)	(1 , 3 , 9 , 21)	(1 , 3 , 21 , 37)	(1 , 3 , 37 , 57)	(1 , 3 , 57 , 81)

III. Conclusion

This paper concerns with the formulation of sequence of Dio-quadruples with property $D\left(k^2+1\right)$. One may search for Dio-quadruples of special numbers with suitable properties.

REFERENCES

[1] L.E. Dickson, *History of the theory of numbers*, Chelsea Publishing Company, Newyork, Vol.2, 1966.

[2] M.A. Gopalan, S. Vidhyalakshmi and A. Kavitha, "Special dio-quadruples with property $D(2)$", *The International Journal of Science & Technoledge*, Vol. 2, Issue 5, pp. 51-52, May 2014.

[3] M.A. Gopalan, G. Sumathi and S. Vidhyalakshmi, "Special $D(k^2+1)$ Dio-Quadruple involving Jacobsthal Lucas and Thabit-ibn-kurrah numbers", *International Journal of Mathematics Trends and Technology*, Vol. 11, No. 2, pp. 77-80, July 2014.

[4] S. Vidhyalakshmi, A. Kavitha and M.A. Gopalan, "Construction of D(5)-dio quadruples", *International Research Journal of Mathematics, Engineering & It*", Vol.1, Issue 4, pp. 16-20, August 2014.

[5] M.A. Gopalan, G. Sumathi and S. Vidhyalakshmi, "Special Dio-Quadruple involving Jacobsthal and Jacobsthal Lucas number with the property $D(k^2+1)$", *International Journal of Math.Sci and Engg.Appls*, Vol. 8, No. III, pp. 221-225, 2014.

[6] M.A. Gopalan, S. Vidhyalakshmi and N. Thiruniraiselvi, "A special diophantine quadruple with property D(17)", *International Journal of Mathematics Trends and Technology*, Vol. 20(2), pp. 110-112, April 2015.

[7] S. Vidhyalakshmi, S. Aarthy Thangam, M.A. Gopalan, "On special sequences of dio-quadruples with property $D(-1)$", *Journal of Information and Computational Science*, Vol. 9, Issue 10, pp.649-658, 2019.