Bianchi type- vi$_0$ Dust Filled Universe with varying Λ(t) in creation field theory of gravitation

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Abstract :We have investigated Bianchi type vi0 cosmological model with the matter dust filled universe with cosmological constant Λ(t) in c- field theory of gravitation. To get deterministic solution, we assumed that Λ = 1/B2 as considered by Chen & Wu, where B is a scale factor and A = B where A and B are metric potentials. The conservation equation $T^{j}_{i;j} = 0$ and $T^{i}_{j} = T^{j}_{i(m)} + T^{j}_{i(c)}$,$T^{j}_{i(m)}$being energy-momentum tensor for matter and $T^{j}_{i(c)}$is the energy momentum tensor for C-field.

We find that creation field (C) increase with time and $\Lambda \sim 1/t^2$which matches with the result obtained by Riess et al. (1998) and Perlmutter et al. (1999). The matter density is constant which is conserved due to the continuous creation of matter and particle horizon does not exist Above we have discussed special cases n = 0, ± 1. Further some physical and geometrical aspects for this model are also studied.

Keywords: Bianchi type vi$_0$, creation field, cosmology, varying cosmological constant.

1. Introduction: In recent years there has been considerable importance in Bianchi type vi$_0$ cosmological models. These models are significant role in the early evolution of the universe.Bianchi type vi$_0$ cosmological models are interesting becausethese models are homogeneous and anisotropic.The uniformly of the field equations formed Bianchi space time helpful in establish models of spatially homogeneous and anisotropic cosmologies. Therefore, these models are to be convenient models of all universes and study of these models are very significant.

Bianchi type-VI_0 magnetizedbarotropic bulk viscous fluid massive string Universe in general relativity have been obtained by Bali, Pradhan and Amirhshchi (1). Bianchi type- VI_0 bulk viscous fluid models with variable gravitational and cosmological constant is also investigated by Verma and Ram (2).String cosmology in bianchi type –VI_0 space-time have been obtained by Biswal, Mahanta, Agarwalla and Adhikary (3). Sadeghi, Amani and Tahmasbi (4) have studied Stability of viscous fluid in bianchi type-VI model with cosmological constant. Accelerating bianchi type- VI_0 bulk viscous cosmological models in Lyra geometry is also investigated by Asgar and Ansari (5).

Khadekar, Samdurkar and Sen (6) have studied Bianchi type –VI cosmological model with quadratic form of time dependent Λ term in general relativity. Bulk viscous bianchi type –IX

string dust cosmological model with time dependent term have been obtained by Parikh, Tyagi and Tripathi (7). Shrimali and Trivedi (8) have studied Decelerating Bianchi type -***Vl_o***universe model with time dependent ***Λ***term.Kaluza-Klein Dust Filled Universe with Time Dependent Λ in Creation Field Cosmology have been obtained by Ghate and Salve (9). Parikh,Tyagi and Tripathi (10) investigated String dust bianchi type –III cosmological model with time dependent cosmological term-Λ.

C-field cosmological model for dust distribution with varying Λ in FRW space-time is also investigated Bali and Saraf (11). Bianchi type-III Dust filled universe with Time dependent Λin C-field cosmology have been obtained by Bali and Saraf (12). Bali and Saraf (13) investigated Bianchi type-I Dust filled universe with Decaying vacuum energy Λ in C-field cosmology.C-field Cosmology with Variable G in the Flat Friedman-Robertson-Walker Model is also investigated by Bali and Tikekar (14). Tyagiand Parikh (15) have studied Bianchi type-Vl_0 cosmological model with barotropic perfect fluid in creation field theory with time dependent Λ.

Cosmological Model with Variable G in C Field Cosmology has been obtained by Bali and Kumawat (16). Chatterjee and Banerjee (17) investigated C-field cosmology in higher dimensions. Singh and Chaubey (18) have studied Bianchi type I, III, V, VI0 and Kantowski-Sachs universes in creation field cosmology. Bali and Kumawat (19) investigated C-field cosmological model with variable G in FRW Space –Time.Kandalkar, Khade and Gawande (20) investigated Bianchi Type VI Bulk Viscous Fluid String Cosmological Model in General Relativity. Bianchi type VI model with cosmic strings in the presence of a magnetic field has been obtained by Saha and Visinescu (21).

Bianchi type VI Anisotropic Dark Energy Model with Varying EoS Parameter is also investigated Saha (22).Tikekar and Patel (23) have studiedSome exact solutions in Bianchi type VI_0 string cosmology. The Bianchi type V magnetized string Dust cosmological model in general relativity has been obtained by Bali and Jain (24).Bali and Sing (25) investigated The Bianchi type V Bulk Viscous Fluid String Dust cosmological model in general relativity. Bali and Upadhaya (26) have studied L.R.S. Bianchi type I string Dust magnetized cosmological models.Bali and Sing (27) have studiedL.R.S. Bianchi type V Bulk Viscous Fluid String Dust cosmological model in general relativity.LRS Bianchi type-V Dust filled universe with Varying Λ(t) in Creation-field theory of Gravitation is also investigated Ghate and Salve (28).

Patil, Bolke and Bayaskar (29) have studied Bianchi type-IX Dust filled universe with Ideal Fluid Distribution in Creation-field. Bali (30) investigated Bianchi type V magnetized string Dust universe with Variable Magnetic Permeability. Bali, Upadhaya and Singh (31) have studied L.R.S. Bianchi type II string Dust cosmological model in general relativity.Bianchi type I String Dust cosmological model with magnetic field in general relativity has been obtained by Bali and Pareek (32). Bali and Sharma (33) investigated Tilted Bianchi type I Dust fluid cosmological model in general relativity. Bianchi type II String Dust Universes is also investigated Ram and Singh (34). Rathore and Bagora (35) have studied Magnetized Dust fluid Tilted Universes for Perfect Fluid Distribution in general relativity. Inhomogeneous Bianchi type VI_0 String Dust cosmological model of Perfect Fluid Distribution in general relativityhas been obtained by Tyagi, Sharma and Chhajed (36).

In this paper, we have observed and investigated theLRS Bianchi type- VI_0 space-time with varying cosmological constant Λ(t) in the creation field theory of gravitation.. For deterministic model, we assumed $\Lambda = 1/B^2$, where B is scale factor. We have also studied and discussed special cases n = 0, $\pm$ 1. The physical aspects and geometrical parameters of the model have been discussed and concluding remarks have been expressed.

2. The Metric and Field Equation

We have considered Bianchi type vi_0 of the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2nx} dy^2 + C^2 e^{-2nx} dz^2 \quad \ldots(1)$$

where A,B and C are functions of time t only and n is a constant.

Einstein field equations by introduction of C-Field are modified by Hoyle and Narlikar (1964a,b,c) as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G \left[T_{i(m)}^j + T_{i(c)}^j \right] + \Lambda g_i^j \quad \ldots(2)$$

The energy-momentum tensor $T_{i(m)}^j$ for perfect fluid and creation field $T_{i(c)}^j$ are given by

$$T_{i(m)}^j = (\rho + p) v_i v^j + p g_i^j \quad \ldots(3)$$

$$T_{i(c)}^j = -f \left(c_i c^j - \frac{1}{2} g_i^j c_\alpha c^\alpha \right) \quad \ldots(4)$$

Here ρ is the energy density of massive particle and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1$. The coupling constant between matter and creation field is greater than zero. It is assumed that creation field C is a function of time only i.e. $C(x,t) = C(t)$.

With the help of equations (3) and (4), the Hoyle-Narlikar field equations (2) for the metric (1) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{n^2}{A^2} = 8\pi G \left(-p + \frac{1}{2} f \dot{c}^2 \right) + \Lambda \quad \ldots(5)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{n^2}{A^2} = 8\pi G \left(-p + \frac{1}{2} f \dot{c}^2 \right) + \Lambda \quad \ldots(6)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{n^2}{A^2} = 8\pi G \left(-p + \frac{1}{2} f \dot{c}^2 \right) + \Lambda \quad \ldots(7)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{n^2}{A^2} = 8\pi G \left(\rho - \frac{1}{2} f \dot{c}^2 \right) + \Lambda \quad \ldots(8)$$

$$\frac{n^2}{A^2} \left(\frac{C_4}{C} - \frac{B_4}{B} \right) = 0 \quad \ldots(9)$$

From equation (9), we get C=lB, & A=B

....(10)

Where$l$ is a constant of integration.

Using equation (10), Field equations (5)-(8) reduce to

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{n^2}{B^2} = 8\pi G \left(-p + \frac{1}{2} f \dot{c}^2 \right) + \Lambda \quad \ldots(11)$$

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{n^2}{B^2} = 8\pi G \left(-p + \frac{1}{2} f \dot{c}^2 \right) + \Lambda \quad \ldots(12)$$

$$2\frac{B_{44}}{B}+\frac{B_4^2}{B^2}-\frac{n^2}{B^2}=8\pi G\left(-p+\frac{1}{2}f\dot{c}^2\right)+\Lambda\ldots(13)$$

$$3\frac{B_4^2}{B^2}-\frac{n^2}{B^2}=8\pi G\left(\rho-\frac{1}{2}f\dot{c}^2\right)+\Lambda\ldots(14)$$

The conservation equation

$$\left(8\pi G T_j^i+\Lambda g_j^i\right)_{;i}=0\ldots..(15)$$

which leads to

$$8\pi\dot{G}\left(\rho-\frac{1}{2}f\dot{c}^2\right)+8\pi G\left[\dot{\rho}-f\dot{c}\ddot{c}+3\rho\frac{B_4}{B}-3f\dot{c}^2\frac{B_4}{B}+3p\frac{B_4}{B}\right]+\dot{\Lambda}=0\ldots..(16)$$

p being an isotropic pressure.

3. Solution of field equation

Following Hoyle and Narlikar (1964a,b,c) for dust distribution, we have taken $p=0$. The source equation of C-field $C^j_{;i}=0$ leads to t C $=t$, for larger r. Thus $\dot{C}$=1.

Using $p=0$, in equation (12), we get

$$2\frac{B_{44}}{B}+\frac{B_4^2}{B^2}-\frac{n^2}{B^2}=4\pi G f\dot{c}^2+\Lambda\ldots(17)$$

Also, using$\dot{C}$=1 in equations s (14) and (17) therein, we have

$$3\frac{B_4^2}{B^2}-\frac{n^2}{B^2}=8\pi G\left(\rho-\frac{1}{2}f\right)+\Lambda \qquad \ldots(18)$$

$$2\frac{B_{44}}{B}+\frac{B_4^2}{B^2}-\frac{n^2}{B^2}=4\pi G f+\Lambda \qquad \ldots(19)$$

Since, Gravitational constant G= constant and $p=0$, equation (16) transfers to

$$8\pi G\dot{\rho}-8\pi G f\dot{c}\ddot{c}+24\pi G\rho\frac{B_4}{B}-24\pi f G\dot{c}^2\frac{B_4}{B}+\dot{\Lambda}=0 \qquad \ldots(20)$$

Solving equations (18) and (19), we get

$$\frac{B_{44}}{B}+2\frac{B_4^2}{B^2}-\frac{(n^2+1)}{B^2}=4\pi G\rho \qquad \ldots(21)$$

To get deterministic solution in terms of cosmic time t, we assume that $\Lambda=\frac{1}{B^2}$. {Chen & Wu (Phys. Rev. D 41:695, 1990)}.

Using $\Lambda=\frac{1}{B^2}$in equation (19), we get

$$2\frac{B_{44}}{B}+\frac{B_4^2}{B^2}-\frac{(n^2+1)}{B^2}=4\pi G f\ldots(22)$$

which again leads to

$$2B_{44}+\frac{B_4^2}{B}-\frac{(n^2+1)}{B}=4\pi G f B\ldots(23)$$

To find the solution of equation (23), Let $\dot{A}$=$F(A)$.

which implies $\ddot{A}=FF'$, where $F'=\frac{dF}{dA}$.

Substituting in equation (23), it leads to

$$\frac{df^2}{dB}+\frac{f^2}{B}=4\pi f G B+\frac{(n^2+1)}{B}\ldots.(24)$$

On integration simplifies to

$$F^2=\frac{4\pi f G}{3}B^2+(n^2+1)\ldots(25)$$

For simplicity, the integration constant taken to be zero.

Using $\dot{A}=F(A)$, equation (25) simplifies to

$$\frac{dB}{\sqrt{B^2+\frac{3(n^2+1)}{4\pi fG}}} = \sqrt{\frac{4\pi fG}{3}}\, dt \qquad \ldots(26)$$

which leads to

$$\frac{dB}{\sqrt{B^2+\alpha^2}} = \beta dt \ldots(27)$$

Where $\alpha = \frac{3(n^2+1)}{4\pi fG}, \quad \beta = \sqrt{\frac{4\pi fG}{3}} \ldots(28)$

Equation (27) on integration gives

$$B = \alpha \sin h\, \beta t$$

In particular for $\beta = 1$, we have

$$B^2 = (n^2+1) sinh^2 t \ldots(29)$$

and $\Lambda = \frac{1}{B^2} = \frac{cosec\, h^2 t}{1+n^2} \qquad \ldots(30)$

Using equations (29) and (30) in equation (18), we have

$$\rho = \frac{1}{8\pi G}\left[3\frac{B_4^2}{B^2} - \frac{(n^2+1)}{B^2} + 4\pi fG\right] \ldots(31)$$

Using equation (29) in metric (1), we get

$$ds^2 = -dt^2 + (n^2+1) sinh^2 t (dx^2 + e^{2nx} dy^2 + e^{-2nx} dz^2) \ldots(32)$$

Case (i): $n = \pm 1$

Equation (25) leads to

$$\frac{dB}{\sqrt{B^2+2}} = dt, \qquad \ldots(33)$$

Where $\frac{4\pi fG}{3} = 1$

Equation (33) on integration gives

$$B = \sqrt{2}\, \sinh t \ldots(34)$$

Using equations (31) and (34) in equation (20), we have

$$\frac{d\dot{c}^2}{dt^2} + (6 \coth t)\dot{c}^2 = 6(\coth t) \qquad \ldots(35)$$

Equation (35) gives $\dot{c}^2 = 1 \qquad \ldots..(36)$

So, we have $\dot{c} = 1 \qquad \ldots(37)$

which agrees with the value used in source equation. Thus, creation field is proportional to time t.

Case (ii): n=0

Equation (24) leads to

$$\frac{dB}{\sqrt{B^2+1}} = dt, \qquad \ldots(38)$$

Where $\frac{4\pi fG}{3} = 1$

Equation (38) on integration gives

$$B = \sinh t \ldots(39)$$

Using equations (31) and (39) in equation (20), we have

$\frac{d\dot{c}^2}{dt^2} + (6\coth t)\dot{c}^2 = 6(\coth t)$ ...(40)

Equation (35) gives $\dot{c}^2 = 1$ .....(41)

So, we have $\dot{c} = 1$...(42)

It is creation field is proportional to time t.

4. Physical Aspects

The Energy mass density (ρ), the cosmological constant (Λ) and the deceleration parameter (q) for the model (32) are given by

$8\pi G\rho = \frac{2\coth^2 t+2}{n^2+1}$...(43)

$\Lambda = \frac{1}{(n^2+1)\sinh^2 t}$....(44)

$q = -\tanh^2 t$...(45)

Where $4\pi f G = 3$ and $\beta = 1$ assumed.

5. Conclusion

The deceleration parameter $\boldsymbol{q} < 0$ pointing that the model (30) describing an accelerating universe.The homogeneous mass density $\boldsymbol{\rho} > 0$ for k = 0, ± 1. The creation field (C) and Spatial volume V increases with time t. The model (46) describing a singularity free model. The cosmological constant $\Lambda \sim 1/t^2$ which matches with the latest Astronomical observations.

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