A detailed investigation and analysis of fuzzy soft graph in social safety network

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Abstract:

Social safety networks are the areas in which a huge number of people are connected. In this paper, a new social safety network called fuzzy soft social safety network (FSSSN) has been introduced based on fuzzy soft graph. For this safety network , centrality, single and multi safety bridges and transfer value of the safety bridges are newly defined and illustrated by examples. In this safety network, the strength of relationship can be graded by different values between 0 and 1, and we have shown that this representation is more realistic. Also, we have introduced a new concept of registration for a new user. So that the chance of fake user may be reduced.

Keywords:

Bridges, centrality, Fuzzy soft graphs, social safety networks.

1. Introduction

- **Now a days social safety networks are new platforms for staying in touch with people in everywhere in the world. These are perfect places of exchanging information of various topics and issues. These help in marketing world, connecting public to clients. These are important tools for public awareness by spreading messages rapidly to a wide audience.**
- **Indeed, social networks are important for information diffusion, e- commerce and e-business, influential players like scientists, innovators, employees, customers, companies etc., effective social and political campaigns, future events, terrorist/criminal network, alumni, etc.,**
- **Facebook , Twitter, LinkedIn, orkut, Research Gate are some types of social networks. Face book was founded by mark zuckerberg in collaboration with his college roommates and fellow students to connect the whole world.**
- **Core Date points Review and updated Wednesday April 24,2019 based upon Face book's official investor relations information are**

    1. **Worldwide, there are over 2.38 billion monthly active users (MAU) as of March 31, 2019. This is an 8 percent increase in Face book MAUs year over year.**
    2. **1.56 billion people on average log onto Face book daily and are considered daily active users (Face book DAU) for March 2019.**

3. Photo uploads total 300 million per day.

4.Number of face book users in India is 260 millions.

- **Twitter is an online social safety networking service and micro blogging service that enables its users to send and read text-based posts.**
- **Twitter updated on 2.April 2019. There is a total of 1.3 billion Twitter accounts, but only 328 million are active (source: blog. Statusbrew.com)**
- **The average Twitter user has 707 followers in a day and handling over one billion search queries per day exist in Twitter.**
- **LinkedIn is a professional social safety networking website founded in 2002 and launched in May 2003. It is mainly used for professional networking.**
- **LinkedIn updated on 29 May 2019. Total number of yearly active LinkedIn users are 630 million and Total number of monthly active linkedIn users are 303 million.**
- **90 million LinkedIn users are senior level influencers and 63 million are in decision making positions.**
- **One million users published on article on LinkedIn.**
- **6th March 2019 updation of Research Gate, Research Gate provides its members with a number of tools to facilitate global scientific collaboration. Researchers can create professional profiles, discuss their topics with specific question and answer forums, share papers, search for jobs and discover conferences in their field. A recent calculation of members shows that Research Gate has so far assembled a user-base of over 15 million researchers from all countries.**
- **Facebook, orkut, Twitter, etc., have many advantages to exchange pictures, videos, comments, etc., The social networks are places to find old or lost friends and to connect new friends. There are places to grow consciousness among internet users. Nowadays, political leaders, ministers, film actors, businessmen etc are spending time in these social safety networks to reach people, specially young generation.**
- **A lot of advantages are available in social safety networks, even then some drawbacks in the current social safety networks are present. These are listed below:**
  - **(i) In all these social safety networks, all social safety units, (people, organizations, etc.) are assumed with same importance. In reality, it is not the case where every unit has the same facility.**
  - **(ii) In the present social safety networks, it is assumed that the strength of friendship between every pair of units are same, but in reality it is not true.**
  - **(iii) In almost all social safety networks, information of one member can not be found by other registered persons if it is not "public". But the other registered persons to the social network may be his/her friend or acquaintance.**
  - **(iv) In almost all social safety networks, the relations are represented, initially, as friendship ("add as friend"). But relationship may be romantic, kinship, friendship, colleagues, parent-child, acquaintance, teacher-student, seller-buyer etc.,**

(v) **Fake profile is one of the big problems of social safety networks. Now, it has become easier to create a fake profile. People often use fake profile to insult, haras someone, involve in unsocial activities, etc.,**

- **These problems motivate us to introduce a new social network model. We make this model using fuzzy soft graph theory.**

2. Preliminaries:

Every kind of social safety group can be represented in terms of safety units, composing this group and relations between these safety units. This kind of representation of a social safety structure is called "Social safety Network". In a social safety network, every unit, usually called "Social safety units" Like a person, an organization, a community, and so on, is represented as a vertex .A relation between two social safety actors is expressed by a edge. So every social safety networks can be represented by a graph.

Definition 2.1:

A graph is a ordered pair G=(V,E) comprising a set V of vertices or nodes together with a set E of edges or lines.

Definition 2.2:

A fuzzy set A on a set X is characterized by a mapping m:X->[0,1], which is called the membership function. A fuzzy set is denoted by A =(X,m).

Definition 2.3:

A fuzzy graph [22] $\xi=(V,\sigma,\mu)$ is a non-empty set V together with a pair of functions σ: V->[0,1] and μ:VxV->[0,1] such that for all $x,y\in V$, $\mu(x,y)\leq \sigma(x)\wedge\sigma(y)$ and μ is a symmetric fuzzy relation on σ. Here $\sigma(x)$ and $\mu(x,y)$ represent the membership value of the vertex x and of the edge (x,y) in ξ respectively.

Definition 2.3

A fuzzy soft graph G=(G*, F, K,A) is a 4- tuple such that,

(a) G*=(V,E) is a simple graph
(b) A is a nonempty set of parameters.
(c) (F,A) is a fuzzy soft set over v,
(d) (K,A) is a fuzzy soft set over E,
(e) (F(a), K(a)) is a fuzzy soft (sub) graph of G* for all $a\in A$. That is $K(a)(xy)=\min\{F(a)(x), F(a)(y)\}$ for all $a\in A$, and $x,y\in V$. The fuzzy soft graph (F(a), K(a)) is denoted by H(a) for convenience.

Definition 2.4:

Degree of a vertex $v \in V$ of a fuzzy soft graph $G=(G^*,F,K,A)$ is $d(V)= \Sigma_{x \neq y} K(a)(xy)$ for all $x \in V$ For the fuzzy soft graph $G=(G^*,F,K,A)$ an edge $K(a)(xy)$, $x, y \in V$ is called strong if $1/2\min\{F(a)(x), F(a)(y)\} \leq K(a)(x\ y)$ and it is called weak otherwise.

Definition 2.5:

As there may be multiple edges between nodes, multi-set is important for this context. A (crisp) multiset over a non-empty set V is simply a mapping $d:V \rightarrow N$ where N is the set of natural numbers.

The membership values of $v \in V$ are denoted as $K^j(a)(xy)$, $j = 1,2,\ldots p$, where $p=\max\{j:K^j(a)(x\ y) \neq 0\}$. So the fuzzy soft multi soft can be denoted as $M=\{F(a)(y), K^j(a)(xy))$, $j=1,2,\ldots p/F(a)y \in V\}$

Definition 2.6:

A multigraph is a graph allowing multiple edges and loops. Let V be a non empty set and $\sigma:V \rightarrow [0,1]$ be a mapping. Also, let $E=\{(x,y), \mu^j(x,y)), j=1,2,\ldots p/(x,y) \in VxV\}$ be a fuzzy multiset of vxv such that $\mu^j(x,y) \leq \sigma(x) \wedge \sigma(y)$ for all $x,y \in V$ and for all $j=1,2,\ldots p$ where $p=\max\{j/\mu^j(x,y) \neq 0\}$.

Then $T=(G^*,F,K,A)$ is denoted as fuzzy soft multigraph where $F(a)(x)$ and $K_{(a)}^j(xy)$ represent the membership value of the vertex x and the membership value of the edge (x,y) in T respectively.

In this paper, a new social safety network called fuzzy soft social safety network (FSSSN) is described.

3.Representation of FSSSN

Here assume all social safety actors as fuzzy soft vertices, and all edges(linkages) as fuzzy soft edges.Moreover,edges(linkages) may be of different kinds. So, represent the social safety network as fuzzy soft multigraph (G^*,F,K,A) where $G^*=(V,E)$, $V=\{v_1,v_2,\ldots v_\lambda\}$, λ is a large positive integer, be a set of social safety units. $F:V \rightarrow [0,1]$ be a mapping and $K=\{((v_i,v_j), K^r(v_i,v_j)), r=1,2,\ldots g/(v_i,v_j) \in VxV\}$ be a fuzzy soft multiset of VxV such that $K^r(v_i,v_j) \leq F(v_i) \wedge F(v_j)$ for all $v_i,v_j \in V$ and for all $r=1,2,\ldots g$ where $g=\max\{r/Kr(v_i,v_j) \neq 0\}$

Without loss of generality, we assumed that all edges (linkages) are of same kind. In this case, unit membership value of a social safety unit is same as unit membership value of a social safety unit, represented in fuzzy soft multi-graph representation but edge (link) membership values are little different.

Then the FSSSN can be represented by a fuzzy soft safety graph FSSSN=(G*,F,K,A) where $F:V\rightarrow [0,1]$ and $K:VxV\rightarrow [0,1]$ are mappings such that $K(v_i,v_j)\leq F(v_i)\wedge F(v_j)$ for all $v_i, v_j \in V$.

The registration for a new user in FSSSN is described below.

3.1 Registration

The first step to join a person to FSSSN is registration. During registration a person has to decide the following steps. Here we add one more setting option, namely automatic authentication, to the existing settings of current social networks.

3.1.1 Basic Settings

The newly registered persons are to add some basic information like name, age, images, gender, etc., All the registered persons will get the basic information of a newly registered person. So the newly registered person is to decide what information he/she wants to give publicly.

3.1.2 Automatic authentication

One of the objective of this step is to avoid creation of fake profile. In registration procedure we introduce a new step called automatic authentication.

In this step, the person has to give information like his/her (i) school/college/university name of his/her educational life. (ii) Location of current job or working place/institution (iii) Frequent visiting institutions, (iv) Best friends names. The information of a new registered person is available for those registered persons whose information will match partially or fully with the new person. Now, some such registered persons are required to identify the new user as a valid person. If some consents (at lest 3 generally) are available, then the new person is considered as full member. This setting is the only way for a new registered person to find new friends initially.

3.1.3 Private Setting

In this settings, the information of the newly registered persons is available for the persons who are selected by the newly registered persons.

3.2 Membership value of a social safety unit

Membership value of a social unit will be assigned between 0 and 1. If a unit has been given membership value 0, the unit is taken as non-member (probably a fake profile). If a unit has membership value 1 the unit is a full member of the safety network. People will be happy to make friends to full members only.

The membership value of a social safety unit is given in the following way.

(i) **When a social safety unit (a person, an institution, etc.) is registered to the social safety networks, initially the membership value of this unit is set to 0.**

(ii) **We divide all the people into two groups. The persons who are working in authentic (viz., government, semi-government, government aided, registered private organization, etc.) Organizations and official C – profiles (contains basic personal information and contact information along with email addresses) are available in their respective institute's web-sites, full into first group, say group – A. These units can be verified by their official e-profiles (viz., email address verification, etc.). This type of units of group-A, are referred as unit by verification.**

Other persons fall into the second group, say group B. These units can not be verified by email address verifications as these persons e-profiles (viz., email addresses , etc) are not available in authentic web-sites. The units can be verified by recognition of registered units. The units of groups-B are referred as units by recognition.

Initially registered social safety units of the group, units by recognition will not be given the scope to make direct friends like "add as friends". There will be an automatic choice of registered persons list by automatic authentication process. The units can send limited messages to only the listed units. After recognition by both ends they can make friendship, kinship, acquaintance, etc. from the list only. The units can not contact other members, if the information, given in "automatic authentication" do not match with each others.

The units by verification initially have the membership value 0. Then a verification code/activation like will be sent to the contact addresses of the social safety units.

(iii) **Units By recognition**

Before the discussion of membership of the social safety units, a correlated term,, recognition number, is defined below. In every society, recognition of each person is measured by some members of the society. In FSSSN, the number of these members as recognition number. It is denoted by n.

The membership value of a social safety unit (p) of the group, unit by recognition, which is conncected with d units, is given by

$$\sigma(p)=\begin{cases} d/n, \text{ if } d=1,2,\ldots n \\ 1, \text{ if } d>n. \end{cases}$$

Obviously, n is recognition number for FSSSN. The social safety network can be planned for units in rural area with small value of n and for urban area with large value of n.

(iv)Units by Verification:

After verification of the personal contact information from their respectively official e-profiles, the units get the full membership value i.e., 1 directly.
Once a unit gets membership value1, it can connect directly to the units with membership value 1 anywhere in the safety network.

3.3 Link membership value

Two social safety units may be connected by any of the social safety relations friendship, kinship, acquaintance, etc. In real situation two friends may be very close, close, etc. So the relations between two units are the elements of a fuzzy set. These relations are designated by membership values.

Sometimes our minor decisions depend on friends decisions. Generally, we accept the decision of a closed friend. The closeness depends on the strength of friendship. So strength of friendship is very important. So membership values of edges(links) between two units are important for a social safety network.

Relations between two persons may be friendship and kinship together. Again "buyer-seller" relation may be termed as acquaintance or even friendship. So two safety units may have multiple relations.

Let $V=\{v_1,v_2,\ldots v_\lambda\}$ be the fuzzy set of registered social safety units in FSSSN Such that $F(P_i)$, $i=1,2,\ldots\lambda$ are the membership values of the social safety units v_i, $i=1,2,\ldots\lambda$ respectively. Edges (Links) between two social safety units form a fuzzy soft multi-set of VxV. Before the definition of membership values of the edges (links) between two social safety units, a co-related term, stable interaction number, is defined below. In FSSSN, stability of relation between two members stability of relation between two members are measures by the number of interactions between them. This number, for stable relation, is defined as the stable interaction number and denoted by the fixed positive integer l.

The edge (link) membership value between $v_i,v_j\in V$ is defined by the mapping $\mu: VxV\rightarrow [0,1]$

Where $K(vi,vj)=\begin{cases} m+1/l\ (F(v_i)\wedge F(v_j)), & \text{if } M\in[0,l-1] \\ F(v_i)\wedge F(v_j) & \text{if } m>l-1 \end{cases}$

For FSSSN, l is stable interaction number, m stands for the number of interactions between two social safety units v_i and v_j per unit of times. The social safety relations are also different types such as friendship, kinship, acquaintance, etc.

Let there be g types of relations in the social safety network. The edge (link) membership value between $v_i,v_j \in V$ be denoted by $K^r(v_i,v_j)$ (Where r represents the type of relations) and defined by

$$K^r(v_i,v_j)= \begin{cases} m_r+1/l\ (F(v_i)\wedge F(v_j)), & \text{if } m_r \in [0,l-1] \\ F(v_i)\wedge F(v_j) & \text{if } m_r > l-1 \end{cases}$$

r=1,2….g, For FSSSN, l is stable intersection number. m_r stands for the number of intersections between two social safety units v_i and v_j per unit of time. If $K^r(v_i,v_j)$ denotes the link membership value of a relation between two persons v_i and v_j we assume that $K^r(v_i,v_j)=0$, r=1…s-1, s+1,….g in FSSSN.

3.3.1 Strong and weak soft edges (links)

All the soft edges (links) called strong soft edges (links) if soft edges (links) membership values are greater than 0.5 and weak otherwise. All decisions will be affected by strong soft edges (links). But only major decisions may be affected by weak soft edges (links).

2. Stopping of fake profiles in FSSSN

Initially registered people in unit by recognition can not make direct friends to his/her choices. A list of people, possibly familiar to his/her, is prepared with the help of his/ her setting, specially in "automatic authentication". If some of the listed persons verify the new registered person, the new member will be a full-member. So creation of a fake profiles is not easy here.

Government/ government sponsored institutes/registered companies have personal web sites. Those web sited contain the names, contact details, information of authorities. If a person wants to register in FSSSN in group, units by verification, then he/she has to give his/her institute's name and web address. Then he/she has to provide his her email address, phone numbers, name and web address. Then he/she has to provide his/her email address, phone numbers, etc., which are available in the web address. Then a verification code/activation edge (link) is send to contact addresses. If the information is correctly verified, then it is almost sure that the person is not fake.

5.An example of FSSSN

A social safety network by 20 people and their inter-relations are shown. We take 7 days as one unit of time, number of recognition friends, n=3 and number of stable interaction between two social safety units, l=2.

5.1 The status of FSSSN After 7 days.

Let after 7 days, the social safety network has 20 registered people. Let $V= \{v_1,v_2,..v_{20}\}$ Initially, these 20 people have membership values 0. v_{15} is assumed as unit by verification and so after verification of contact details,v_{15} will get full membership value 1.

Other units, they can sending some enquiry messages to others, they can make recognition friends. The number of recognition friends is 3 here. After connecting 3 recognition friends, they will get full membership value 1.

The list of people with connected friends and number of interactions after 7 days are shown in Table 1.

Table 1:

List of people and their connected people. The number within parenthesis represents number of interactions after 7 days.

Person	Connected people with number of interactions
v_1	$v_2(0)$, $v_4(2)$
v_2	$v_1(0)$
v_3	$v_4(2)$
v_4	$v_1(2)$, $v_3(2)$, $v_5(2)$,$v_{17}(2)$
v_5	$v_4(2)$
v_6	X
v_7	$v_{11}(3)$
v_8	$v_9(1)$, $v_{10}(0)$, $v_{20}(1)$
v_9	$v_8(1)$
v_{10}	$v_8(0)$, $v_{11}(1)$
v_{11}	$v_7(3)$, $v_{10}(1)$, $v_{13}(0)$
v_{12}	X
v_{13}	$v_{11}(0)$
v_{14}	X
v_{15}	X
v_{16}	X
v_{17}	$v_4(2)$, $v_{19}(0)$
v_{18}	X
v_{19}	$v_{17}(0)$
v_{20}	$v_8(1)$

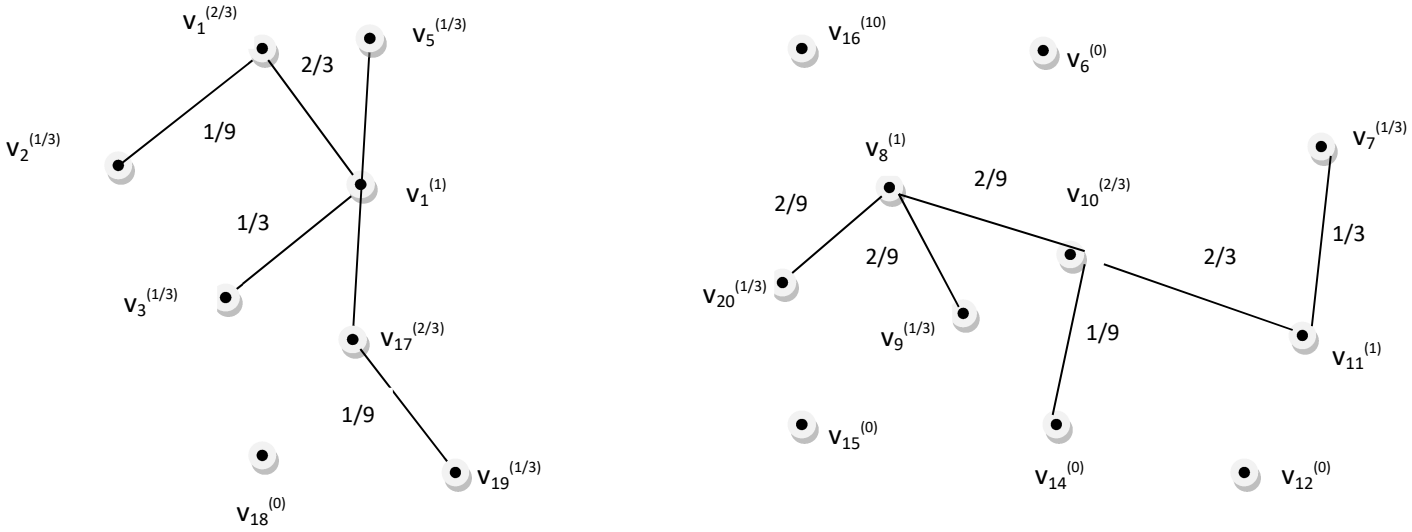

Figure:1 Status of FSSSN after 7 days

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The number in parenthesis of v_1 is unit membership value and number adjacent to edge (link) represents the edge (link) membership value.

5.2 The status of FSSSN after 14 days.

We observe the edges(linkages) of same 20 people after 14 days.

Here is the statistics of the people. After 14 days, i.e., after two units of time, the numbers of interactions between two units are shown in table 2.

The edge (link) membership value is calculated as follows. We see that the number of interaction after 14 days between v_1 and v_2 is 2. So the number of interaction per unit time, m is 1. Now $K(v_1,v_2)= m+1/l\ (F(v_1)\hat{}F(v_2))$

Table 2:

List of people and their connected people. The number within parenthesis represents number of interactions after 14 days.

Person	Connected people with number of interactions
v_1	$v_2(2)$, $v_4(4)$
v_2	$v_1(2)$, $v_4(4)$
v_3	$v_4(4)$
v_4	$v_1(4)$, $v_2(4)$, $v_3(4)$, $v_5(4)$, $v_{17}(3)$
v_5	$v_4(4)$
v_6	$v_8(1)$
v_7	$v_8(1)$,$v_{10}(1)$, $v_{11}(5)$
v_8	$v_6(1)$, $v_7(1)$, $v_9(3)$,$v_{10}(1)$,$v_{20}(3)$
v_9	$v_8(3)$, $v_{11}(6)$, $v_{14}(2)$
v_{10}	$v_7(1)$, $v_8(1)$, $v_{11}(6)$, $v_{13}(4)$, $v_{14}(1)$
v_{11}	$v_7(5)$, $v_9(6)$, $v_{10}(6)$, $v_{12}(1)$, $v_{13}(1)$, $v_{14}(1)$
v_{12}	$v_{11}(1)$, $v_{13}(3)$
v_{13}	$v_{10}(4)$, $v_{11}(1)$, $v_{12}(3)$, $v_{14}(2)$
v_{14}	$v_9(2)$, $v_{10}(1)$, $v_{11}(1)$, $v_{13}(2)$
v_{15}	$v_{13}(4)$, $v_{20}(4)$
v_{16}	X
v_{17}	$v_4(3)$, $v_{19}(1)$
v_{18}	$v_{19}(7)$, $v_{20}(3)$
v_{19}	$v_{17}(1)$, $v_{18}(7)$,$v_{20}(2)$
v_{20}	$v_8(3)$, $v_{18}(3)$, $v_{19}(2)$

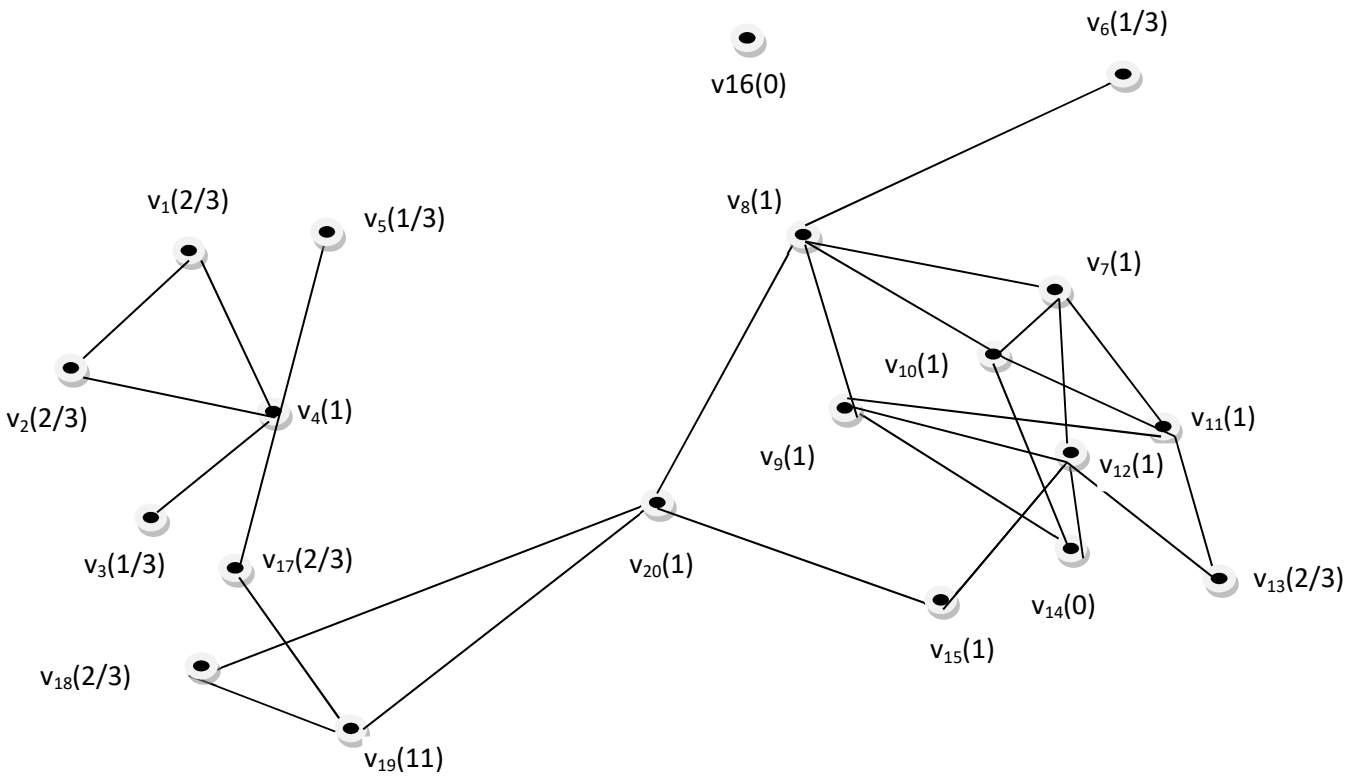

Figure 2: Status of FSSSN after 14 days.

Table 3:

List of edge and edge membership values after 14 days.

Edge	Edge membership values
$K(v_1,v_2)$	0.44
$K(v_1,v_4)$	0.667
$K(v_2,v4)$	0.667
$K(v_3,v_4)$	0.333
$K(v_4,v_{17})$	0.556
$K(v_6,v_8)$	0.167
$K(v_7,v_8)$	0.5
$K(v_7,v_{10})$	0.5
$K(v_7,v_{11})$	1
$K(v_8,v_9)$	0.833
$K(v_8,v_{10})$	0.5
$K(v_8,v_{20})$	0.833
$K(v_{13},v_{15})$	1
$K(v_9,v_{11})$	1
$K(v_9,v_{14})$	0.667
$K(v_{10},v_{11})$	1
$K(v_{10},v_{13})$	1
$K(v_{10},v_{14})$	0.5
$K(v_{11},v_{12})$	0.333
$K(v_{11},v_{13})$	0.5
$K(v_{11},v_{14})$	0.5
$K(v_{12},v_1)$	0.556
$K(v_{13},v_{14})$	0.667
$K(v_{17},v_{19})$	0.333
$K(v_{18},v_{19})$	0.667
$K(v_{18},v_{20})$	0.556
$K(v_{19},v_{20})$	0.667
$K(v_{15},v_{20})$	1

6. Centrality of a social safety unit in FSSSN.

Centrality is one of the most studied concepts in social safety network analysis, numerous measures have been developed, including degree centrality, closeness, betweenness eigenvector centrality, information centrality, the rush index, etc.

Degree centrality measures the direct edge of social safety unit to the others, betweenness centrality measures the number of paths between any pair of social unit safety s through the social safety unit. In all kinds of network, central persons are more valuable than others. They can send messages to more people, can collect more information. Degree centrality measures the number of direct friends. So it does not measure the number of connected people by a path. It may be noted that friend of a friend in facebook shares information to the person. So betweeness centrality is also useful. But friend of friend does not share so much information like the direct friend. So importance of the edge will gradually decrease from a person to another person by a connected path.

In FSSN, if a unit v_f is directly connected with the safety unit v, then we say that v_f is distance -1 friend of v. The set of all distance -1 friends of v be denoted by $d_1(v)$. That is $d_1(v)=\{v_i \in V; v_1$ is a distance friend of v$\}$

Similarly if there is a shortest path (i.e., minimum number of edges) between v and v_f containing K edges or links, then v_f is a distance-K friend ofvP. That is $d_k(v_0=\{v_i \in V; v_i$ is a distance –k friend of v$\}$

Now, let $d_k(v)=d_k(v)-d'_{k-1}(v_0$, where K=2,3,…. and $d_1'(v)=d_1(v)$

It is natural that the distance -1 friends are more important than distance -2 friends, distance-2 friends are more important than distance-3 friends and so on. The linguistic term "more important" can be represented by weights. Let w_k, $0 \le \omega_k \le 1$ be the weight which represents the importance between the distance –k friends. The weight gradually decreases if the distance between the friends increases. Thus $\omega_1 \ge \omega_2 \ge \ldots \ge \omega_k \ge \ldots\ldots$.

Let $u_1(=v_i)$, $u_2,u_3,\ldots u_k(=v_j)$ be the vertices on the path betweenv_i and v_j. We define fuzzy distance $D_f(v_i,v_j)$ between v_i and v_j along this path as $D_f(v_i,v_j) = \sum_{l=1}^{k-1} K(u_l,u_l+1)$.

In a network, it may be observed that there are multiple paths between two vertices. In FSSN, we consider those paths of same length whose fuzzy distance D_f is maximum. If there are K edges in this path of maximum fuzzy distance, then we denote this distance by D_f^k i.e., $D_f^k(v_i,v_j)$ represents the fuzzy distance between the vertices v_i and v_j in FSSSN along a certain path containing exactly K edges. We assumed that a social unit v has at most distance –v friends.

Now we define the centrality C(v) of a social unit v of FSSSN as follows

$$C(v) = \sum_{u_1 \in d_1^1(v)} w_1 D_f^1(v,u_1) + \sum_{u_2 \in d_1^1(v)} w_2 D_f^2(v,u_2) + \ldots\ldots + \sum_{u \ v \in d_1^1(v)} w_v D_f^v(v,u_v)$$

In this measurement, the importance of closed friend is given more than the next to close friends and gradually decreases the furthest friends. The importance are introduced by incorporating the weight w_i, for distance –i friend, i=1,2,….

A FSSSN of 20 people after 14 days is shown in figure 2. The edge membership values are shown in Table 3. In the definition of centrality of a social unit, v can be taken as fixed for a social safety networks. Here we assumed that v = 3 and measure the centrality of social safety units. Here take w_1=1 and w_{iH}= (1/2) w_i, i=1,2,…

6.1 Centrality of v_1

Here $d_1(v_1)$= $\{v_2,v_4\}$=$d_1'(v_1)$

$d_2(v_1)$=$\{v_4,v_3,v_{17},v_5\}$, $d_2(v_1)$=$d_2(v_1)$-$d_1'(v_1)$

=$\{v_3,v_{17},v_5\}$

$d_3(v_1)$=$\{v_{17},v_{19}\}$, $d_3'(v_1)$=$d_3(v_1)$-$d_2'(v_1)$=$\{v_{19}\}$

$$\sum_{u_1 \in d_1^1} v_1 \; D_f'(v_1,U_1)=K(v_1,v_2)+k(v_1,v_4)$$

$$=0.44+0.667$$

$$=1.107$$

Similarly,

$$\sum_{u_2 \in d_1^1(v1)} D_f^2(v_1,u_2) = D_f^2(v_1,v_3) + D_f^2(v_1,v_5) + D_f^2(v_1,v_{17})$$

$$= \{ K(v_1,v_4) + k(v_4,v_3)\} + \{ K(v_1,v_4) + k(v_4,v_5)\} + \{ K(v_1,v_4) + k(v_4,v_{17})\}$$

$$=\{0.667+0.333\}+ \{0.667+0.333\}+\{0.667+0.556\}$$

$$=3.223$$

Also $$\sum_{u_3 \in d_2^1(v1)} D_f^3(v_1,u_3) = D_f^3(v_1,v_{19})$$

$$=\{ K(v_1,v_4) + k(v_4,v_{17}) + k(v_{17},v_{19})\}$$

$$=\{0.667+0.556+0.333\}$$

$$=1.556$$

The centrality of the safety unit v_1 is given by

$$C(v_1) =\sum_{u_1 \in d_1^1(v1)} D_f^1(v_1,u_1) + \sum_{u_2 \in d_2^1(v1)} 0.5\, D_f^2(v_1,u_2) + \sum_{u_3 \in d_3^1(v1)} 0.25\, D_f^3(v_1,u_3)$$

$$= 3.1075$$

Similarly, we calculate centralities of other social safety units.

6.1.1 Important observation

In figure 2, there are three paths between, v_{15} and v_9 of length 3. They are v_{15}-v_{20}-v_8-v_9-v_{15}-v_{13}-v_{11}-v_9 and v_{15}-v_{13}-v_{14}-v_9. The fuzzy distance of first path is 2.66, the fuzzy distance of second path is 2.5 and that of third path is 2.334. Thus to calculate the centrality of v_{15}. We have to choose the path v_{15}-v_{20}-v_8-v_9 as its fuzzy distance is most calculating we get $C(v_{15})$=10.1795.

Again the centrality of v_4,$C(v_4)$ is 3.7785. But v4 has 5 direct friends (i.e., distance -1 friends) and v_{15} has 2 direct friends. So centrality in FSSSN depends on both direct friends and strong distance –P friends, here v>1.

7 Social bridges in FSSSN

Sometimes two friends are connected by a third friends. If the third friends is lost then two friends may be disconnected. Here the third friend is called a broker or structural hole between two friends. Similarly, if two groups of people are connected by a single person, the person is broker or structural hole between two groups. Opinion and behavior are more homogenous within the group, so people connected across groups are more familiar with alternative ways of thinking and behaving. People who stand near the holes in a social safety structure are at higher chance of having good ideas. Social safety bridges are constructed by at least three safety units, one from each group and other is broker. We denote a bridge of three units v_i,v_j,v_k by $B(v_i,v_j,v_k)$.

7.1 Social safety bridges of more than three units.

Sometimes, one person can not establish a connection between two groups of people. But he has a friend who is connected to one person of another group. If these two persons are connected, two groups will be connected and a safety bridge will be formed by four social safety units. Also this kind of safety bridge can be extended to more people. (Figure 3)

For example, v_1 is a student of a university A. v_2 is a friend of v_1. v_3 is brother of v_2. Again v_3 has friend v_4 who is a student of university B. So the two universities are connected by a safety bridge of four safety units.

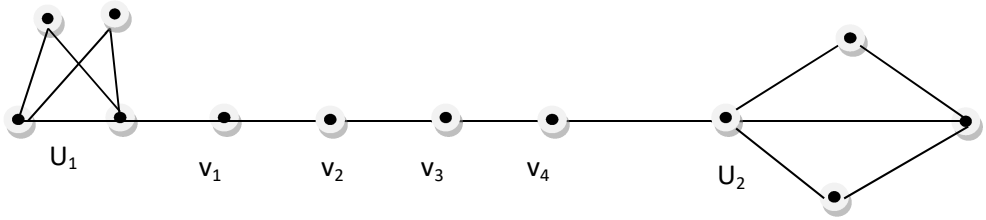

Figure3: A social safety bridge constructed by more than three safety units.

7.2 Strong and weak safety bridges

If a broker or structural hole is strongly connected to two groups of people, connected to each other, the transfer of information will be very speedy. But, if the connection is weak, information flow will not be fast. So not only broker or safety bridge is an important safety unit in a social safety network, but also its strength. Depending on the strength, a safety bridge can be classified as strong and weak safety bridges.

Definition 7.2.1:

If the edge (link) membership values of all the edges (links) in a safety bridge are greater than 0.5, then the safety bridge is called strong safety bridge. On the other hand, if edges membership value of at least one edge is less than or equal to 0.5, then the safety bridge is called weak safety bridge.

Let a safety bridge be constructed by social safety units v_1,v_2,v_3 between two groups of people A_1 (Connected with v_1) and A_2 (Connected with v_3). Let the edge(link) membership value between v_2 and v_1 be $K(v_2,v_1)$ and that of between v_3 and v_2 be $K(v_3,v_2)$. The strong and weak safety bridges are depicted in figure 4.

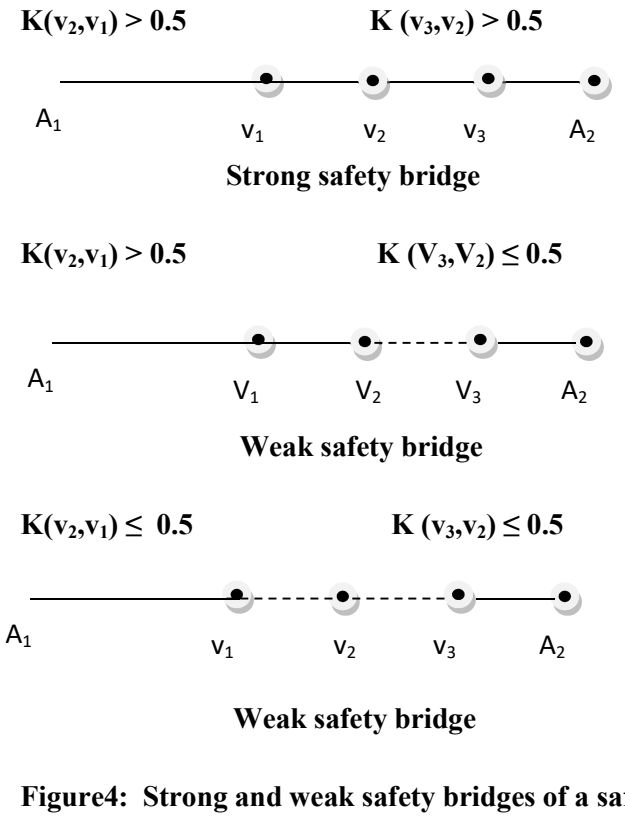

Figure4: Strong and weak safety bridges of a safety network.

7.3 Multi safety bridges

In real field, two different groups of people want to interact with each other. For example, an Indian people's groups want to interact with the culture of an Australian people's group and vice-versa. If the two groups have a safety bridge, then the safety bridge is the only way to get the information from each other. Sometimes, it may be happened that the social safety units of the safety bridge do not share the complete information or not have enough faith on the groups. So another safety bridge will help to transfer the information between groups. In this situation, alternative safety bridge is required to get the complete and correct information. If two or more disjoint safety bridges exist between two groups or safety units, then these safety bridges are called multi- safety bridges (see figure 5)

Here each safety bridge is called a branch of the multi- safety bridge. We denote the multi- safety bridge by $\{B_1,B_2,\ldots B_k\}$ where each B_i, i=1,2,…k is a safety bridge.

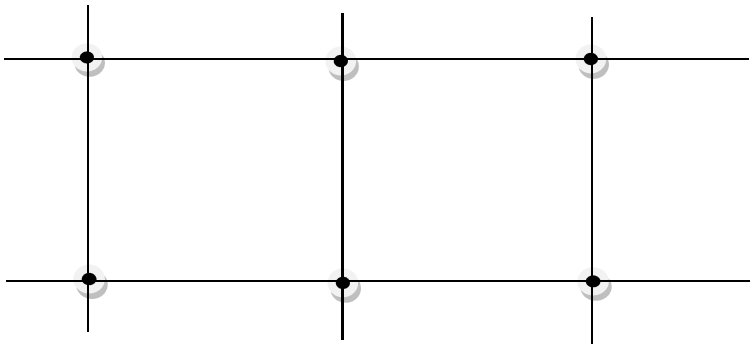

Figure5: Multi- Safety Bridge of a safety network

In modern day, concept of Safety Bridge is not so helpful like multi- safety bridge and strong safety bridges. Nobody can believe single person or single social safety unit. Even a safety bridge is strong, multi- safety bridge is required to correct and transfer fast information. Also, multi- safety bridge network is more reliable than the single safety bridge network. In bigger area of people, social capital of brokers in multi- Safety Bridge will not reduce so much unless number of safety bridges in a multi- safety bridge are too large. Sometimes, people confuse about which safety bridge is good in terms of transfer of information. To remove the confusion, we have introduced the transfer value of safety bridges in FSSSN. If a safety bridge is connected to a person with large centrality value of a grouop,the safety bridge will transfer large amount of information to another group. Similarly, if a safety bridge is connected to a person with small centrality value of a group, Safety Bridge will transfer little information to another group.

7.4 Transfer value of safety bridges in FSSSN

Let a safety bridge B be constructed by social safety units, v_i, i=1,2,…z between two groups of people A, connected with v_1 and B, connected with vz. Let the edge (link) membership value between P_i and P_j be $K(P_i,P_j)$.Let minimum of the edge membership value of the edges in B be m. The transfer value of B be denoted by T(B) and defined by the value $T(B)=mxC(v_1)\hat{}C(v_z)$ where C(v) denotes the centrality of a person in FSSSN. For multi- safety bridge the transfer value is defined by the sum of the transfer values of each safety bridges.

Figure 6:(a): A safety bridge B_1 and its transfer value.

$C(v_1) = 4.2$ $C(v_4) = 5.6$

$T(B_1)$= 0.4x$C(v_1)$^$C(v_4)$=0.4x4.2=1.68 , $T(\{B_1,B_2\})$-1.68+4.03=5.71

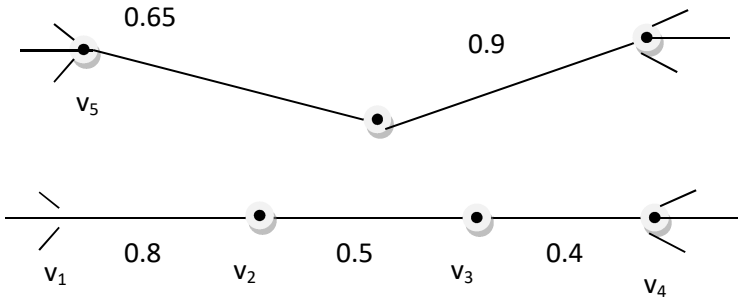

Figure (b): A multi safety bridge $\{B_1,B_2\}$ and its transfer value.

Example: Let a safety bridge B_1 in FSSSN constitutes social safety units v_1,v_2,v_3,v_4 where v_1,v_4 are social units connected to two different groups of people figure 6(a). Let $K(v_1,v_2)$=0.8, $K(v_2,v_3)$=0.5, $K(v_3,v_4)$=0.4 let $C(v_1)$=4.2, , $C(v_4)$=5.6 so the transfer value of B_1, $T(B_1)$=0.4x$C(v_1)$^$C(v_4)$=0.4x4.2=1.68

In figure 6(b) a multi- safety bridge $\{B_1,B_2\}$ is shown with social units v_1,v_2,v_3,v_4,v_5,v_7 and $K(v_1,v_2)$=0.8, $K(v_2,v_3)$=0.5, $K(v_3,v_4)$=0.4, $K(v_5,v_6)$=0.65, $K(v_6,v_7)$=0.9.

$C(v_1)$=4.2, $C(v_4)$=5.6, $C(v_5)$=6.2, $C(v_7)$=8.2 For the multi safety bridge, transfer value $T(\{B_1,B_2\})$ is 1.68+4.02=5.71

Using transfer value people can choose the safety bridge for faster transmation in FSSSN.

8 Conclusions

This study opens a new area of social safety networks. A social safety network model FSSSN has been introduced with the help of fuzzy soft graph. In the proposed FSSSN, the registered persons will not be able to create fake profile easily. So the people identify the valid persons with their membership values. If a person in unit by verification category gets full membership value, the person can not be fake generally. But, if a person in unit by recognition category gets full membership value, then there is a chance that the profile is fake. If a group of persons is involved to recognize each other, then fake profile can be created. But, if one of the units is a valid unit (i.e., unit by verification), then there is little chance to create a fake profile.

Centrality of a person in FSSSN has been define. Here fuzzy distance of connected friends of different distance is considered for measurement of centrality. Between two person, if there are multiple paths of same distance, the path whose fuzzy distance is most, is considered for calculation. To measure the centrality we considered the fuzzy soft path whose fuzzy soft distance is maximum. A different measure of centrality be given by considering the strength of a fuzzy soft path.

Transfer value of safety bridge is defined here. So which safety bridge is better for information transfer, can be identified by the formula of transfer value, centrality of persons and transfer value of safety bridges will be helpful in marketing world. In future, this model will help to represent telecommunication systems by fuzzy soft graphs.

Also edge (link) prediction in social safety networks will be introduced in future as an extension of this study.

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