THE EQUATION OF CONTINUITY AND JUNCTION THEOREM IN FLUID DYNAMICS

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ABSTRACT

In this research paper we have described some special conditions of the equation of continuity in fluid dynamics. If we take $\nabla.(\rho q) = \rho\nabla.q + \nabla\rho.q$ then the equation of continuity becomes $\frac{d(\log\rho)}{dt} + \nabla.q = 0$. We have given a theorem, which we named as junction theorem of fluid dynamics. This theorem is based on the concept of the law of conservation of mass. In this research paper we have derived a general concept, how to find the equation of continuity when the fluid particle, describes circle in plane.

Keywords: Equation of continuity, Junction theorem, Law of conservation of mass, Logarithmic form of continuity equation

1. Introduction

Law of conservation of physical quantity is a fundamental rule of nature. The physical quantity mass is one of such physical quantity which obeys the rule of conservation. The conservation of mass is a very important principle which is used widely in various branches of sciences, specially in engine sciences. Conservation principle of mass says that mass is neither created nor destroyed but remain same before and after the action. During fluid flow total mass does not change but it remains conserved. This principle of conservation of mass in fluid dynamics is known as law of continuity or equation of continuity.

If ρ is the fluid density, q is the velocity of fluid flow then differential form of the continuity equation [1] with respect to time t will be, $\frac{\partial\rho}{\partial t} + \nabla.(\rho q) = 0$.

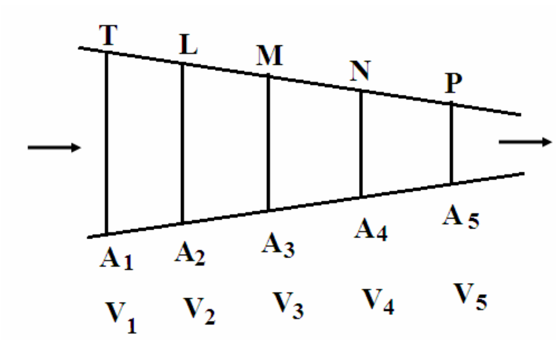
2. Corollary - If we put $\nabla.(\rho q) = \rho \nabla q + \nabla \rho . q$

then the equation of continuity

$\frac{\partial \rho}{\partial t} + \nabla.(\rho q) = 0$ will be as follow,

$\frac{d(\log \rho)}{dt} + \nabla . q = 0$. It is known as logarithmic form of continuity equation in fluid dynamics.

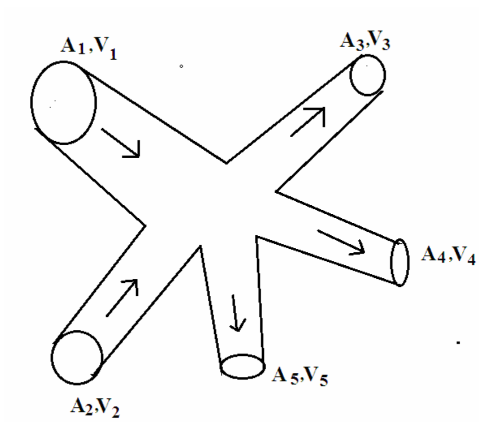
3. Corollary :- For an incompressible fluid flow through a pipe whose cross sectional area is not fixed, then



$A_1V_1 = A_2V_2 = A_3V_3 = A_4V_4 = A_5V_5 = \ldots\ldots$

Here (A_1, V_1), (A_2, V_2), (A_3, V_3), (A_4, V_4), (A_5, V_5) are area of cross section of pipe and velocity of given pipe at different arbitrary points T, L, M, N and P.

4. Junction theorem (For incompressible fluid) – Total mass of fluid entering from different cross sections of a system at junction becomes equal to the total masses of fluid leaving at different cross sections of the given system at that junction.



Here, $A_1 V_1 + A_2 V_2 = A_3 V_3 + A_4 V_4 + A_5 V_5$

$$\therefore \left(\sum_{i=1}^{n} A_i V_i \right)_{\text{entering}} = \left(\sum_{k=1}^{m} A_k V_k \right)_{\text{leaving}}$$

As i, k, n, m are integers and A_i, A_k as well as V_i, V_k are areas and velocities of fluid flow at different cross sections of the system.

5. Theorem:- The product of the speed and cross sectional area remains constant along a stream filament of a fluid in steady motion.

6. Corollary [Based on streamline flow] If speed of fluid flow is same everywhere then streamlines will be equations of straight lines.

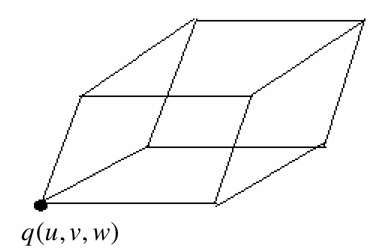
7. Corollary:- If $\nabla.(\rho q) = \rho \nabla.q + \nabla \rho.q$ then the equation of conservation of mass $\frac{\partial \rho}{\partial t} + \nabla.(\rho q) = 0$ will be written as $\frac{\partial \rho}{\partial t} + \rho \nabla.q + \nabla \rho.q = 0$. Further we can also write the above equation of continuity as,

$$\frac{D\rho}{Dt} + \rho \nabla.q = 0$$

8. Corollary:- If we have incompressible and heterogeneous fluid, then ρ will be a function of x, y, z and t so, $\frac{D\rho}{Dt} = 0$ as well as $\nabla.q = 0$.

9. Corollary:- If we have an incompressible and homogeneous fluid then $\frac{\partial \rho}{\partial t} = 0$ and the equation of continuity will be $\nabla.(\rho q) = 0$ or $\nabla.q = 0$.

10. How to apply the conservation of mass in fluid flow when the fluid particle makes circle in different planes. Let us suppose a parallelepiped and there be a particle of fluid flow at any one corner of this parallelepiped with density ρ.



The edges which are considered as arcs will be taken as $\lambda\delta\alpha, \mu\delta\beta$ and $v\delta y$. Here, λ, μ, v are radii in different planes of a fluid particle, while $\delta\alpha, \delta\beta, \delta y$ are angles between radii in different planes.

Now we suppose as follow

length of elements (arcs)	$\lambda\delta\alpha$	$\mu\delta\beta$	$v\delta y$
Component of velocity	u	v	w.

Hence, the rate of the excess of the flow-in over flow-out along first length, as $-\lambda\delta\alpha\frac{\partial}{\lambda\partial\alpha}(\rho u\mu\delta\beta v\delta y)$ in other words, (- first length) $\times$ derivative with respect to the first length of the product (density $\times$ velocity in the first direction $\times$ product of remaining lengths.) In the same way we can obtain the excess of the flow-in over the flow-out as follow, (-second length) $\times$ derivative with respect to the second length of the product (density $\times$ velocity in the second direction $\times$ product of remaining lengths.). Again we have (- third length) $\times$ derivative with respect to the third length of the product (density $\times$ velocity in the third direction $\times$ product of remaining lengths.)

Thus the total mass of fluid = density $\times$ product of the three edges of the element

$= \rho\lambda\delta\alpha\mu\delta\beta v\delta y$ and the rate of increase of mass of fluid

$$= \frac{\partial}{\partial t}(\rho\lambda\delta\alpha\mu\delta\beta\, v\delta y)$$

Now according to the law of conservation of mass we have,

$$\frac{\partial}{\partial t}(\rho\lambda\delta\alpha\mu\delta\beta\, v\delta y) = -(\lambda\delta\alpha)\frac{\partial}{\lambda\partial\alpha}(\rho u\mu\delta\beta v\delta y) - (\mu\delta\beta)\frac{\partial}{\mu\delta\beta}(\rho\, \mathrm{v}\lambda\delta\alpha v\delta y)$$

$$-(v\delta y)\frac{\partial}{v\delta y}(\rho w\lambda\delta\alpha\mu\delta\beta)$$

After simplification we get as $\frac{\partial}{\partial t} + \nabla.(\rho q) = 0$ the equation of continuity in fluid dynamics. This method can be used in the derivation of the equation of continuity when the fluid particle describes circle in different planes.

11. Conclusions:- In this research paper we have obtained logarithmic form of the continuity equation of the fluid flow. It will be the special case of the equation of continuity in fluid dynamics. We have got the junction theorem which is based on the concept of law of conservation of fluid dynamics. We have also obtained the general form of continuity equation of fluid dynamics when fluid particle describes circle. Thus through this concept we can easily find the equation of continuity whenever fluid particle describe circle in plans.

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